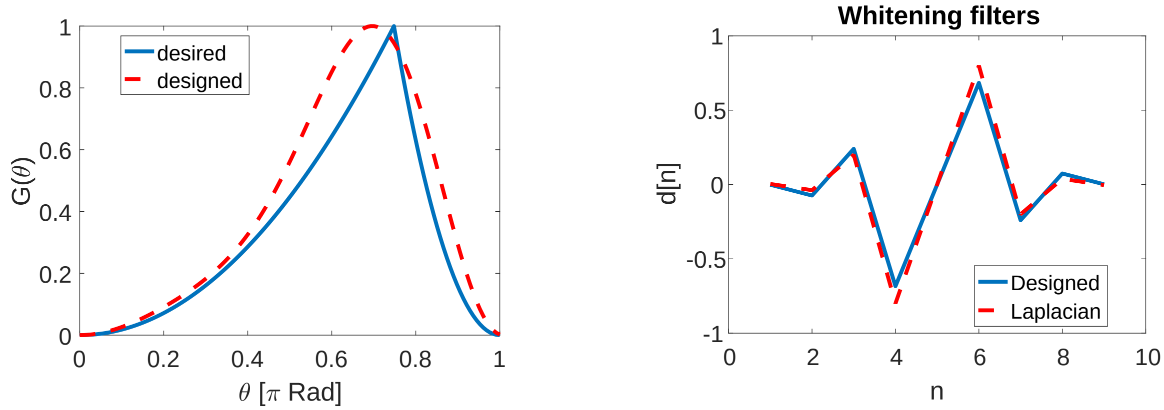

(a) (b)

Figure 1: Example of a Laplacian filter ($G(\theta) = |\theta|^2$) design. The designed filter (a) is comprised of the frequency squared response (for $\theta \leq \frac{3}{4}\pi$) and the complementary response ($\theta > \frac{3}{4}\pi$), to satisfy the symmetry condition ($G(\pi) = 0$). The discrepancy between the designed and desired responses is caused by the small filter order. The resulting filter (b) is similar to the Laplacian used in [1] (dashed red).

SUPPLEMENTARY: WHITENING FILTER DESIGN

In [1], a 9-tap square root of a Laplacian filter is used to whiten the signal. This Laplacian has an approximated frequency response squared proportional to $|\omega|^2$ for a 1D frequency ω. It is therefore suitable as a model of natural images ($|\omega|^{-2}$). In our case, we estimate the exponent as a floating-point number and therefore require a more refined method of whitening filter design.

The desired response of the filter is designed as follows. Given α, an estimated exponent, the whitening filter is

$$G(\omega) = |\omega|^{\alpha/2}. \tag{1}$$

This is due to the fact that

$$x_2 = x_1 * h \Rightarrow S_{X_2}(\omega) = S_{X_1}(\omega) |G(\omega)|^2,$$

and to perform whitening of x_1 with $P_{X_1}(\omega) \propto \omega^{-\alpha}$, the filter itself should have a frequency response with an exponent $\alpha/2$. There exist several alternatives for designing such a filter. We choose the filter to be as similar as possible to a Laplacian, specifically: anti-symmetrical, with compact support and odd-valued. We thus choose a Type III linear-phase FIR filter.

To create a Type III filter, its periodicity should be taken into account; it should satisfy $G(0) = G(\pi) = 0$. Therefore, instead of using (1) directly, we design the desired response for the range $\theta_1 \in \left[0, \frac{3}{4}\pi\right]$ and complete the response in the range $\theta_2 \in \left(\frac{3}{4}\pi, \pi\right)$ via a first order polynomial, $p(\theta)$ so that the response is continuous and $G(\pi) = 0$. The filter is then designed with the same length as in [1] (9-tap) with standard least-squares FIR filter design methods. We note that using Type III FIR filters we can reconstruct almost exactly the Laplacian filter used in [1] when designing a filter with desired exponent $\alpha = 2$ (Fig. 1).

REFERENCES

[1] Amit Goldstein and Raanan Fattal. Blur-kernel estimation from spectral irregularities. In *Eur. Conf. Comput. Vis.*, pages 622–635. Springer Berlin Heidelberg, oct 2012.